International Journal of Knowledge and Language Processing *www.ijklp.org*
Volume 3, Number 2, **2012** KLP International ©2012 ISSN 2191-2734
pp.20—29

A Fractional Differential Approach to Low Contrast Image Enhancement

CHEN Qing-li [1,*], HUANG Guo [2] and ZHANG Xiu-qiong [3]

[1] School of Computer Science
Leshan Normal University
Leshan, Sichuan, 614004, China
cctcop75@yahoo.com.cn

[2] School of Computer Science
Leshan Normal University
Leshan, Sichuan, 614004, China
huangguoxuli@yeah.net

[3] School of Computer Science
Leshan Normal University
Leshan, Sichuan, 614004, China
zxq_03@tom.com

[*] Corresponding author

Received April 2012; revised August 2012

ABSTRACT. *In order to improve the image contrast, a 2-D isotropic fractional differential algorithm for contrast enhancement is constructed, and its structure and parameters on 8 directions are presented. This nonlinear filter algorithm is implemented on various low-contrast digital images and the contrast enhancement of image is controlled by fractional order. The capability of fractional differential mask for contrast enhancement is dicussed. The Experiments and analysis show that the proposed method has excellent feedback for enhancing contrast of dark images, it enhances edge information effectively and reveals more detailed information than HE and SSR method for low-contrast images.*

Keywords: Image enhancement; Fractional Difference; Contrast Enhancement; Fractional Differential Mask

1. Introduction. Contrast enhancement is one of the most important issues of image processing and analysis, and plays an important role in image processing, pattern recognition, computer vision, and etc. It is believed that contrast enhancement is a crucial step to sharpen the low contrast image. Image enhancement is employed to transform an image on the basis of the psychophysical characteristics of human visual system.

Many algorithms for achieving contrast enhancement have been developed. Histogram

Equalization(HE) is the most commonly used method due to its simplicity and comparatively better performance on almost all types of images[1]. Arici et al[2] present a general histogram modification framework for image contrast enhancement. Yoon and Song[3] introduce a generalized histogram method with the fractional count. Polesel et al[4] present a unsharp masking for contrast enhancement. Yi Wan et al[5] present a wavelet-based method. Starck et al[6] propose a curvelet transform based method for contrast enhancement. Gilboa et al[7] generalize the linear and nonlinear scale spaces to complex diffusion processes by incorporating the free Schrodinger equation and present a regularized shock flter for image enhancement. Kundra et al[8] propose a fuzzy logic control based flter for image enhancement.

Recently years, fractional calculus(FC) has become more and more important in foundational research and engineering application. FC provides derivation and integration of functions to non-integer order. Fractional differential is an effective mathematical method for dealing with fractal problems[9-11]. Applying Fractional differential to image processing is a burgeoning subject branch under discussion[12-13]. PU et al have proven that fractional differential-based methods can furthest preserve the low-frequency contour feature in those smooth areas, and non-linearly keep high-frequency marginal feature in those areas that gray-level changes greatly, and also enhance texture details in those areas that gray-level does not change evidently[12-14].

Our interest in this work is to develop a fractional-based technique for contrast enhancement for gray-scale images, and demonstrate the capability of the fractional differential mask by varying the fractional differential order.

The outline of the paper is as follows: in section 2, we briefly review the necessary theoretical background of the R-L based fractional differential, next, the fractional differential-based filter operator is constructed in section 3. Section 4 present the experimental results of the proposed method on three real image. A conclusion is considered in section 5.

2. Theoretic Analyzing. In this section, we review briefly review the necessary theoretical background of the Riemann-Liouville(R-L) based fractional differential. Let $f(x)$ be defined in the interval $[a,b]$ (where a and b can even be infinite) and $\nu > 0$. The left Riemann-Liouville fractional integral of order $\nu < 0$, of a function f , is defined as[9-11,15-16]

$$ {}_aI_x^{\nu} f(x) = \frac{1}{\Gamma(\nu)} \int_a^x (x-\xi)^{\nu-1} f(\xi) d\xi \quad , \tag{1} $$

where a, x are the limits of the definition, and $\Gamma(x)$ is the gamma function of x .

Suppose f is a continuous function in $[a,b]$, the left R-L fractional derivative of order ν, is given by

$$ {}_aD_x^{\nu} f(x) = \frac{d^n}{dt^n} {}_aI_x^{n-\nu} f(x) = \frac{1}{\Gamma(n-\nu)} \frac{d^n}{dt^n} \int_a^x (x-\xi)^{n-\nu-1} f(\xi) d\xi \tag{2} $$

In fact, if f is n times differentiable, the formula[9-10]

$$
{}_aD_x^\nu f(x)=\sum_{k=0}^{n-1}\frac{(x-a)^{k-\nu}f^{(k)}(a)}{\Gamma(k-\nu+1)}+\frac{1}{\Gamma(n-\nu)}\int_a^x\frac{f^{(n)}(\xi)}{(x-\xi)^{\nu-n+1}}d\zeta \tag{3}
$$

defines an analytic function of ν for $\mathrm{Re}(\nu)<n$ and thus provides a valid analytic continuation of formula (1). In particular, when $a=0$, $n=1$ and $0<\nu\le 1$, (3) can be rewritten as

$$
{}_aD_x^\nu f(x)=\frac{x^{-\nu}f(0)}{\Gamma(1-\nu)}+\frac{1}{\Gamma(1-\nu)}\int_0^x\frac{f^{(1)}(\xi)}{(x-\xi)^\nu}d\zeta \cong\frac{1}{\Gamma(1-\nu)}\left[\frac{f(0)}{x^\nu}+\sum_{k=0}^{N-1}\int_{kx/N}^{(kx+x)/N}\frac{f'(x)}{(x-\xi)^\nu}d\xi\right] \tag{4}
$$

3. The Proposed Algorithm.

In this section, we give the discrete expression of fractional derivative and construct the fractional differential mask for image contrast enhancement.

3.1 Discrete Expression of Fractional Derivative. In this part, we concentrate on description of numerical method used for solution of equation (3). Without loss of generality, suppose $a=0$, then the spatial domain is $\Omega=[0,x]$. We divide Ω into the uniform mesh with $N+1$ nodes $x_j=j\Delta x$ for $j=0,1,\Lambda,N$, where $\Delta x=x/N$, and the time-nodes are

$$
f_0=f(0), f_1=f(x/N), \Lambda, f_j=f(jx/N), \Lambda, f_N=f(x) \tag{5}
$$

When N is big enough, by approximating, we obtain

$$
\begin{aligned}
\int_{kx/N}^{(kx+x)/N}\frac{f'(x)d\xi}{(x-\xi)^\nu} &\cong \frac{f\left(\dfrac{kx+x}{N}\right)-f\left(\dfrac{kx}{N}\right)}{\Delta x}\int_{kx/N}^{(kx+x)/N}\frac{d\xi}{(x-\xi)^\nu} \\
&\cong \frac{f\left(\dfrac{kx+x}{N}\right)-f\left(\dfrac{kx}{N}\right)}{-(1-\nu)(\Delta x)^\nu}\left\{[N-(k+1)]^{1-\nu}-[N-k]^{1-\nu}\right\}
\end{aligned} \tag{6}
$$

Then, taking (6) into (1), we have

$$
{}_aD_x^\nu f(x)\cong\frac{1}{\Gamma(2-\nu)}\left\{\begin{array}{l}1^{1-\nu}f_N+\left(2^{1-\nu}-2\cdot 1^{1-\nu}\right)f_2+\Lambda+\\ \left[(N-k+1)^{1-\nu}+(N-k-1)^{1-\nu}-2\cdot(N-k)^{1-\nu}\right]f_k\\ +\Lambda+\left[(N-1)^{1-\nu}-N^{1-\nu}+(1-\nu)N^{-\nu}\right]f_0\end{array}\right\}. \tag{7}
$$

Equation (7) is the numerical form of fractional differential, which simplifies fractional differential to multiplication and addition. Here, the numerical algorithm of fractional differential is noted as ***CRL operator***. Now recalling the definition of first order-differential, it is evident that the operator covering $\nu=0$ and $\nu=1$ (when $\nu=0$, ${}_aD_x^\nu f(x)\equiv f(x)$, and $\nu=1$, ${}_aD_x^\nu f(x)\cong f'(x)$).

We observe that equation (7) can be seen as a discrete representation of convolution of function f and coefficient function C, and equation (7) can be expressed as

$$
{}_aD_x^\nu f(x)\cong\sum_{i=0}^{N}C_i\cdot f_i=C*f=f*C \quad . \tag{8}
$$

These $N+1$ nonzero coefficients $C_i(i=0,\Lambda,N)$ are functions with respect to the fractional order ν. And the coefficients are

$$
\begin{aligned}
&c_0 = 1/\Gamma(2-\nu), c_1 = \left(2^{1-\nu} - 2\cdot 1^{1-\nu}\right)/\Gamma(2-\nu),\ \Lambda \\
&c_k = \left[(N-k+1)^{1-\nu} + (N-k-1)^{1-\nu} - 2(N-k)^{1-\nu}\right]/\Gamma(2-\nu),\ \Lambda \\
&c_n = \left[(N-1)^{1-\nu} - N^{1-\nu} + (1-\nu)N^{-\nu}\right]/\Gamma(2-\nu)
\end{aligned}
\quad . \tag{9}
$$

It is easy to prove that sum of the coefficients is not equal to zero which is a prominent difference between fractional and integral differential.

3.2 Fractional Differential Mask. As for $n_x \times n_y$ digital gray image $f(x,y)$, the shortest distance of 2-D image on x-and y-coordinate is one pixel. Thus, we have $\Delta x = 1$. Then, the approximate differences of fractional partial differentiation on negative x-and y-coordinate, can be respectively expressed as

$$
\frac{\partial^\nu f(x,y)}{\partial x^\nu} \cong \frac{1}{\Gamma(2-\nu)} \left\{ \begin{aligned} &1^{1-\nu} f(x,y) + \left(2^{1-\nu} - 2\cdot 1^{1-\nu}\right) f(x-1,y) + \Lambda + \\ &\left[(N-k+1)^{1-\nu} + (N-k-1)^{1-\nu} - 2\cdot(N-k)^{1-\nu}\right] f(x-k,y) \\ &+\Lambda + \left[(N-1)^{1-\nu} - N^{1-\nu} + (1-\nu)N^{-\nu}\right] f(x-n,y) \end{aligned} \right\} \tag{9}
$$

and

$$
\frac{\partial^\nu f(x,y)}{\partial y^\nu} \cong \frac{1}{\Gamma(2-\nu)} \left\{ \begin{aligned} &1^{1-\nu} f(x,y) + \left(2^{1-\nu} - 2\cdot 1^{1-\nu}\right) f(x,y-1) + \Lambda + \\ &\left[(N-k+1)^{1-\nu} + (N-k-1)^{1-\nu} - 2\cdot(N-k)^{1-\nu}\right] f(x,y-k) \\ &+\Lambda + \left[(N-1)^{1-\nu} - N^{1-\nu} + (1-\nu)N^{-\nu}\right] f(x,y-n) \end{aligned} \right\}. \tag{10}
$$

To obtain the fractional differential on the eight symmetric directions and make the fractional differential masks have anti-rotation capability, the other 6 fractional differential masks, which are $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$, $7\pi/4$ respectively, are implemented. The fractional differential mask is shown in Fig. 1.

$\frac{C_n}{sum}$	0	0	$\frac{C_n}{sum}$	0	0	$\frac{C_n}{sum}$
0	$\ddots$	0	$\vdots$	0	$\iddots$	0
$\vdots$	0	$\frac{C_1}{sum}$	$\frac{C_1}{sum}$	$\frac{C_1}{sum}$	0	$\vdots$
$\frac{C_n}{sum}$	$\cdots$	$\frac{C_1}{sum}$	$\frac{8C_0}{sum}$	$\frac{C_1}{sum}$	$\cdots$	$\frac{C_n}{sum}$
$\vdots$	0	$\frac{C_1}{sum}$	$\frac{C_1}{sum}$	$\frac{C_1}{sum}$	0	$\vdots$
0	$\iddots$	0	$\vdots$	0	$\ddots$	0
$\frac{C_n}{sum}$	0	0	$\frac{C_n}{sum}$	0	0	$\frac{C_n}{sum}$

FIGURE 1. fractional differential Mask

Where C_0 is the coefficient on interested pixel $f_0 = f(x,y)$, the other blank pixels are filled with zeros, and the sum is

$$sum(x,y) = 8 * \left(\sum_{i=0}^{n} C_i \right) \tag{11}$$

It is not difficult to prove that the sum of mask coefficients in Fig. 1 is not equal to zero, by which implies that the filter response of fractional differential is not zero in image region where the gray values of pixels are constant or changes little. This feature is necessary to enhance the contrast, especially for remote sensing images and medical images.

The windows size of mask can be an arbitrary odd number, and a larger window can improve the accuracy of fractional differential, but the computational complexity increases at the same time. The algorithm of fractional differential mask is also taking airspace filtering of mask convolution.

4. Experiment Results and Analysis. This section presents the experimental results of applying the proposed and commonly used methods for local contrast enhancement of details in a real image. For ease of calculation, only the first three coefficients are considered in this paper, and the 3 coefficients are $c_0 = 1/\Gamma(2-\nu)$, $c_1 = (2^{1-\nu}-2)/\Gamma(2-\nu)$, $c_2 = (3^{1-\nu} - 2 \cdot 2^{1-\nu} + 1)/\Gamma(2-\nu)$ respectively.

Three representative images are shown in figures (2), (3) and (4). First is dark living-room image, second one is university image, the third one is a book image. All images are 256 grey scale.

Fig. 2(c)- Fig. 2 (f) are the results of applying the proposed method on a dark university image with different values of differential order. Fig.3- Fig.4 are the results obtained by the HE method, SSR(Single-scale Retinex) method[17-18], fractional methods in [12,14] and the proposed method, respectively. For the comparison purpose, we use the contrast of GLCM (Gray-level Co-occurrence Matrix) in four directions as the capability of contrast enhancement techniques. And Tab.1- Tab.3 are the contrasts of GLSM in four directions.

Fig. 2(a) is a low-contrast, dark image. After employing the proposed method with different differential order, the abundant complex textural detail information is becoming more clear-cut, and the image becomes clearer. Higher values of differential order v give more clearly detailed image and more visible features. And the contours of the buildings, trees and ground are more and more distinct with increasing v, and the contrast of GLCM in four directions also increase with the increase of the differential order(as shown in Tab.1).

Comparing the images in Fig. 3(a)- Fig. 3(f), we can see that the contrast enhanced significantly through the entire image in Fig. 3(d). The contour of the two men become clear, the face of the shoes, cans, bottle et al. are better enhanced. From Table 1, we also observed that the proposed method produce biggest contrast values than HE and SSR method.

Based on the results shown in Fig. (3)- Fig. (4), we can see that the proposed method is more effective than other methods. The proposed method can not only non-linearly preserves the low-frequency contour feature in the smooth area to the furthest degree, but also non-linearly enhance high-frequency marginal information in those areas where gray-scale changes frequently, thus the proposed method improves contrast of small details and makes the image clearer.

From the above experiments, we can conclude that the proposed approach outperforms HE and SSR methods.

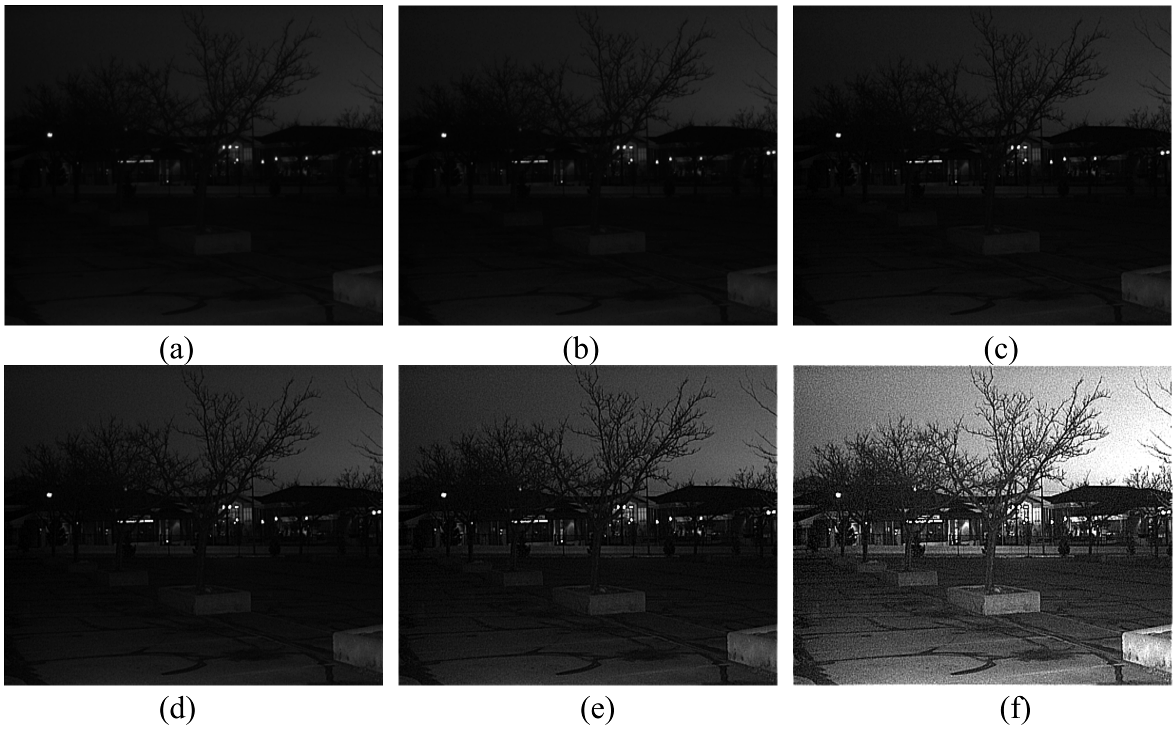

FIGURE 2. result of applying the proposed method on a dark university image with different values of differential order ν. (a) original image. (b)-(f) output images with ν=0.1, ν=0.3, ν=0.5, ν=0.7, ν=0.9, respectively.

TABLE 1. contrast of GLCM in 4 directions with different order

angle / ***order***	0^o	45^o	90^o	135^o
0.0	0.1041	0.1312	0.1224	0.1344
0.1	0.1211	0.1497	0.1412	0.1536
0.3	0.1614	0.2021	0.1898	0.2044
0.5	0.2359	0.3013	0.2848	0.3042
0.7	0.4833	0.5847	0.5464	0.5935
0.9	2.3668	2.7638	2.5605	2.7979

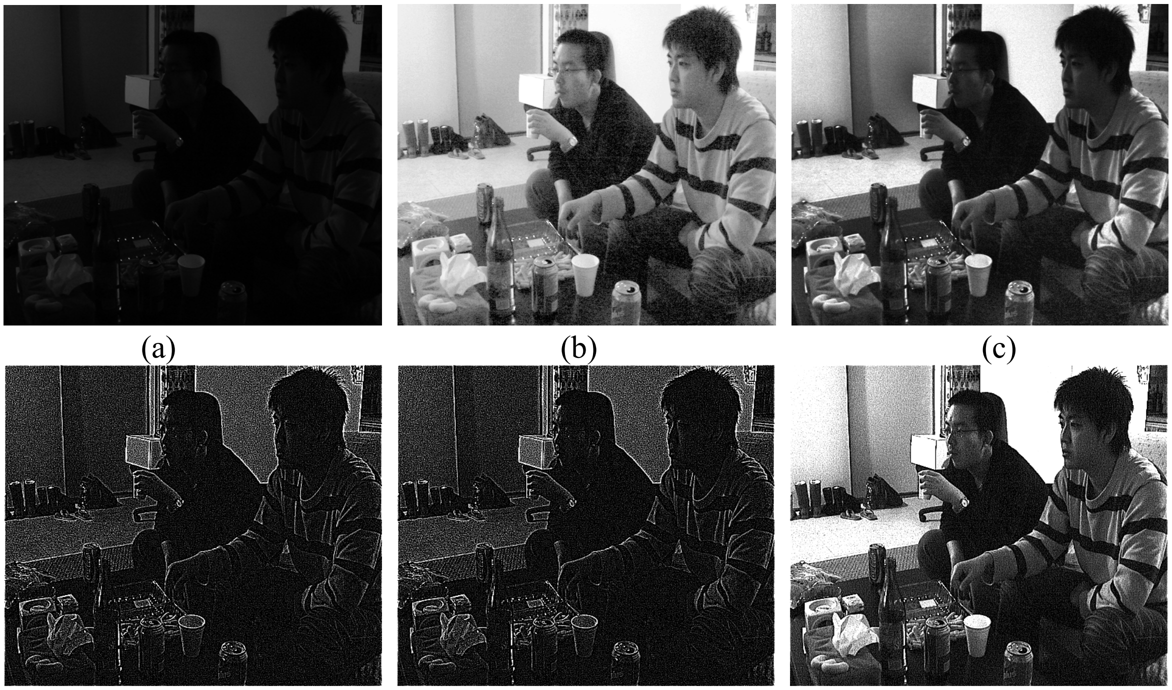

FIGURE 3. Contrast enhancement capability comparison (a) The original living-room image with size 512*512. (b) enhancing result of (a) by HE method. (c) enhancing result of (a) by SSR method. (d) enhancing result of (a) by the method[14] with ν=0.94. (e) enhancing result of (a) by the method[12] with ν=0.94. (f) enhancing result of (a) by the proposed method with ν=0.92.

TABLE 2. contrast of GLCM in 4 directions

angle / ***method***	0°	45°	90°	135°
original	0.0855	0.1004	0.0666	0.0990
HE	0.7987	0.9873	0.6583	0.9429
SSR	0.9987	1.2314	0.7475	1.2330
literature [14]	2.0906	2.2399	1.7780	2.2117
literature [12]	2.1069	2.2549	1.7991	2.2268
the proposed method	2.6334	2.9547	2.0736	2.9049

FIGURE 4. Contrast enhancement capability comparison (a) The original book image with size 720*576. (b) enhancing result of (a) by HE method. (c) enhancing result of (a) by SSR method. (d) enhancing result of (a) by the method[14] with ν=0.53. (e) enhancing result of (a) by the method[12] with ν=0.53. (f) enhancing result of (a) by the proposed method with ν=0.53

TABLE 3. contrast of GLCM in 4 directions

angle / ***method***	0^o	45^o	90^o	135^o
original	0.3741	0.5295	0.3972	0.5441
HE	0.7329	1.0508	0.7583	1.0489
SSR	0.7484	1.0525	0.7559	1.0796
literature [14]	1.0246	1.2989	1.0387	1.3091
literature [12]	1.0521	1.3176	1.0614	1.3327
proposed method	1.4838	1.9151	1.4484	1.9322

5. **Conclusion.** In this paper, we present a R-L fractional differential mask, which can enhance both contrast and texture of image. The method can control the degree of contrast enhancement with fractional differential order ν. And computer simulation results using a real image have shown superiority of the proposed algorithms comparing with conventional algorithms. The method can be implemented further on real remote sened images, Infrared Image, etc.

As a future work, we are planning to use adaptive fractional order for image enhancement.

Acknowledgment. This work is partially supported by the National Natural Science Foundation of China (No. 61201438), and the Scientific Research Fund of Leshan Normal University (No. Z1224), the authors would like to thank all the reviewers for their valuable suggestions.

REFERENCES

[1] R. C. Conzalez and R. E. Woods, Digial Image Processing. Prentice-Hall, New Jersey, 2001.

[2] Tarik Arici, Salih Dikbas and Yucel Altunbasak, A histogram modication framework and its application for image contrast enhancement, IEEE Transactions on Image Processing, vol. 18, no. 9, pp. 1921-1935, 2009.

[3] Byoung-Woo Yoon, Woo-Jin Song, Image contrast enhancement based on the generalized histogram. Journal of Electronic Imaging, vol. 16, no. 3, pp. 1-8, 2007.

[4] Andrea Polesel, Giovanni Ramponi, and V. John Mathews, Image Enhancement via Adaptive Unsharp Masking, IEEE Transactions on Image Processing, vol. 9, no. 3, pp. 505-510, 2000.

[5] Yi Wan, Dongbin Shi, Joint Exact Histogram Specication and Image Enhancement through the Wavelet Transform, IEEE Transactions on Image Processing, vol. 16, no. 9, pp. 2245–2250, 2007.

[6] Jean-Luc Starck, Fionn Murtagh, Emmanuel J. Cands, and David L. Donoho, Gray and Color Image Contrast Enhancement by the Curvelet Transform, IEEE Transactions on Image Processing, vol. 12, no. 6, pp. 706-717, 2003.

[7] Guy Gilboa, Nir Sochen and Yehoshua Y. Zeevi, Image Enhancement and Denoising by Complex Diffusion Processes, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 26, no. 8, pp. 1020-1036, 2004.

[8] Harish Kundra, Aashima and Monika Verma, Image Enhancement Based On Fuzzy Logic, International Journal of Computer Science and Network Security, vol. 9, no. 10, pp. 141-145, 2009.

[9] Hilfer R. Applications of Fractional Calculus in Physics. Word Scientific Press, Singapore, 2000.

[10] J. Sabatier, O. P. Agrawal and J. A. Tenreiro Machado, Advances in Fractional Calculus, Theoretical Developments and Applications in Physics and Engineering, Springer, New York, 2007.

[11] Igor Podlubny, Fractional Differential Equations, Academic Press, New work, 1999.

[12] PU Yi-Fei, Zhou Ji-Liu and Yuan Xiao, Fractional Differential Mask: A Fractional Differential-Based Approach for Multiscale Texture Enhancement, IEEE Transaction on Image Processing, vol. 19, no. 2, pp. 491–511, 2010.

[13] PU Yi-Fei, WANG Wei-Xing, ZHOU Ji-Liu, et al, Fractional differential approach to detecting textural features of digital image and its fractional differential filter implementation, Science in China Series F: Information Sciences, vol. 51, no. 9, pp. 1319–1339, 2008.

[14] PU Yi-Fei, Application of Fractional Differential Approach to Digital Image Processing, Journal of Sichuan University (Engineering Science Edition), vol. 39, no. 3, pp. 124–132, 2007.

[15] Keith B. Oldham, Jerome Spanier, The Fractional Calculus: Theory and Applications of

Differentiation and Integration to Arbitrary Order, Academic Press, New York, 1974.
[16] Kenneth S. Miller, Bertram Ross, An introduction to the fractional calculus and fractional differential equations, Wiley, New York, 1993.
[17] Brian Funt, Florian Ciurea, John McCann, Retinex in Matlab, Journal of Electronic Imaging, January, vol. 13, no. 1, pp. 48-57, 2004,
[18] Meylan L, Susstrunk S, Bio-inspired Image Enhancement for Natural Color Images, *Proc*. of IS&T/SPIE Electronic Imaging 2004: Human Vision and Electronic Imaging IV, 5292, pp. 46-56, 2004.